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# RADIO MEAN D-DISTANCE LABELING OF SOME GRAPHS

**T. Nicholas***

**K.JohnBosco***

**M.Antony***

**Abstract**

A Radio Mean D-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to ℕ such that for two distinct vertices u and v of G, $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G)$, where $d^D(u, v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio mean D-distance number of f, $rmn^D(f)$ is the maximum label assigned to any vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the minimum value of $rmn^D(f)$ taken over all radio mean D-distance labeling f of G. In this paper we find the radio mean D-distance number of some well known graphs.

**Keywords:D-distance, Radio D-distance coloring, Radio D-distance number, Radio mean D-distance, Radio mean D-distance number.**

**AMS Subject Classification :** 05C12, 05C15, 05C78.

* **Department of Mathematics, St. Jude's College, Thoothoor, K.K.Dist, TN, India**

## Introduction

By a graph G = (V, E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimum radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labeling (multi-level distance labeling) can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by Hale [6]. Chartrand et al. [2] introduced the concept of radio labeling of graph. Chartrand et al. [3] gave the upper bound for the radio number of Path. The exact value for the radio number of Path and Cycle was given by Liu and Zhu [10]. However Chartrand et al. [2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of Cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O.Togni [8]. M.M.Rivera et al. [20] gave the radio number of $C_n \times C_n$, the cartesian product of $C_n$. In [4] C.Fernandez et al. found the radio number for complete graph, star graph, complete bipartite graph, wheel graph and gear graph. M.T.Rahim and I.Tomescu [16] investigated the radio number of Helm Graph. The radio number for the generalized prism graphs were presented by Paul Martinez et.al. in [11].

The concept of D-distance was introduced by D. Reddy Babu et al. [17, 18, 19].If u, v are vertices of a connected graph *G*, the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s)$

+ deg(v) + deg(u) +$\sum$ deg($w$) where the sum runs over all intermediate vertices *w* of *s* and $\ell$(s) is the length of the path. The *D*-distance, $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{\ell(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in *G*.

In [12], we introduced the concept of Radio D-distance. The radio D-distance coloring is a function f : V(G) → $\mathbb{N} \cup \{0\}$ such that $d^D(u, v) + |f(u) - f(v)| \geq \text{diam}^D(G) + 1$. It is denoted by $rn^D(G)$. A radio D-distance coloring $f$ of G is a minimum radio D-distance coloring if $rn^D(f) = rn^D(G)$, where $rn^D(G)$ is called radio D-distance number.

Radio mean labeling was introduced by R. Ponraj et al [13,14,15]. A radio mean labeling is a one to one mapping $f$ from V(G) to $\mathbb{N}$ satisfying the condition

$$d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \text{diam}(G). \qquad (1.1)$$

for every u, v ∈V(G). The span of a labeling $f$ is the maximum integer that f maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labelings of the graph G. The condition (1.1) is called radio mean condition.

In this paper, we introduce the concept of radio mean D-distance number. A radio mean D-distance labeling is a one to one mapping $f$ from V(G) to $\mathbb{N}$ satisfying the condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \text{diam}^D(G). \qquad (1.2)$$

for every u, v ∈V(G). The span of a labeling $f$ is the maximum integer that $f$ maps to a vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the lowest span taken over all radio mean D-distance labelings of the graph G. The condition (1.2) is called radio mean D-distance condition. In this paper we determine the radio mean D-distance number of some well-known graphs. The function f:V(G) → $\mathbb{N}$ always represents injective map unless otherwise stated.

## 2 . Main Result

**Theorem 2.1.**

The radio mean D-distance number of a complete graph, $rmn^D(K_n) = n$.

**Proof .**

Since $diam^D(G) = d^D(u, v)$ for any $u, v \in V(K_n)$ the condition (1.2) implies $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1$ for all $u, v \in V(K_n)$. Since f is injective it follows that $rmn^D(K_n) \leq n$. Since $|V| = n$, $rmn^D(K_n) \geq n$. □ Hence the result.

**Theorem 2.2.**

The radio mean D-distance number of a complete bipartite graph

$$rmn^D(K_{m,n}) \leq \begin{cases} 3\left(\frac{m}{2}\right) + 2n - 1 \text{ if } m \text{ is even } m \geq 2, n \geq 2. \\ 3\left(\frac{m-1}{2}\right) + 2n - 1 \text{ if } m \text{ is odd } m \geq 3, n \geq 3. \end{cases}$$

**Proof .**

Since $n \geq m$, $n + 2m + 2 \leq m + 2n + 2$ which implies $diam^D(K_{m,n}) = m + 2(n+1)$. Let $\{v_1, v_2, v_3, \ldots, v_m\}$ and $\{u_1, u_2, u_3, \ldots, u_n\}$ are the partite sets. We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = m + 2n + 3$,

for every pair of vertices (u, v) where $u \neq v$.

**Case 1.** $m(\geq 2)$ is even and $n \geq 2$.

Define the function $f$ as $f(v_i) = \left(\frac{m}{2}\right) + n + i - 1$, $1 \leq i \leq m$, $f(u_i) = 3\left(\frac{m}{2}\right) + n + i - 1$, $1 \leq i \leq n$.

For $(v_i,u_j)$, $d^D(v_i, u_j) + \left\lceil \frac{f(v_i)+f(u_j)}{2} \right\rceil \geq m + n + 1 + \left\lceil \frac{\left(\frac{m}{2}\right) + n + i - 1 + 3\left(\frac{m}{2}\right) + n + j - 1}{2} \right\rceil \geq m + 2n + 3$.

For $(v_i,v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq m + 2n + 2 + \left\lceil \frac{\left(\frac{m}{2}\right) + n + i - 1 + \left(\frac{m}{2}\right) + n + j - 1}{2} \right\rceil \geq m + 2n + 3$.

And for $(u_i,u_j)$,

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 2m + n + 2 + \left\lceil \frac{3\left(\frac{m}{2}\right) + n + i - 1 + 3\left(\frac{m}{2}\right) + n + j - 1}{2} \right\rceil \geq m + 2n + 3$.

Therefore, $f(u_n) = 3\left(\frac{m}{2}\right) + 2n - 1$ is the largest label.

**Case 2.** m( ≥ 3) is odd and n ≥ 2.

Define the function $f$ as $f(v_i) = \left(\frac{m-1}{2}\right) + (n-1) + i - 1, 1 \le i \le m$ and

$f(u_i) = 3\left(\frac{m-1}{2}\right) + (n-1) + i, 1 \le i \le n$. For $(v_i, u_j)$,

$$d^D(v_i, u_j) + \left\lceil \frac{f(v_i)+f(u_j)}{2} \right\rceil \ge m + n + 1 + \left\lceil \frac{\left(\frac{m-1}{2}\right) + (n-1) + i - 1 + 3\left(\frac{m-1}{2}\right) + (n-1) + j}{2} \right\rceil \ge m + n + 3.$$

For $(v_i, v_j)$

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \ge m + 2n + 2 + \left\lceil \frac{\left(\frac{m-1}{2}\right) + (n-1) + i - 1 + \left(\frac{m-1}{2}\right) + (n-1) + j - 1}{2} \right\rceil \ge m + 2n + 3.$$

And for $(u_i, u_j)$,

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge 2m + n + 2 + \left\lceil \frac{3\left(\frac{m-1}{2}\right) + (n-1) + i + 3\left(\frac{m-1}{2}\right) + (n-1) + j}{2} \right\rceil \ge m + 2n + 3.$$

Therefore, $f(u_n) = 3\left(\frac{m-1}{2}\right) + 2(n-1) + 1$ is the largest label.

$$\text{Hence, } rmn^D(K_{m,n}) \le \begin{cases} 3\left(\frac{m}{2}\right) + 2n - 1 \text{ if } m \text{ is even } m \ge 2, n \ge 2. \\ 3\left(\frac{m-1}{2}\right) + 2(n-1) + 1 \text{ if } m \text{ is odd } m \ge 3, n \ge 3. \end{cases}$$

□

**Note.** When m = n, $rmn^D(K_{m,n}) \le \begin{cases} 7\left(\frac{m}{2}\right) - 1 \text{ if } m \text{ is even} \\ 7\left(\frac{m-1}{2}\right) + 1 \text{ if } m \text{ is odd} \end{cases}$

**Theorem 2.3.**

The radio mean D-distance number of a path, $rmn^D(P_n) \le \begin{cases} 2, & n = 2. \\ 4(n-2), & n \ge 3. \end{cases}$

**Proof .**

It is obvious that $diam^D(P_n) = 3(n-1)$. Let $V(P_n) = \{v_1, v_2, v_3, \dots, v_n\}$. Define the function $f$ as $f(v_1) = 3n - 7$, $f(v_2) = 3n$, $f(v_3) = 3n + 1$, $f(v_{i+3}) = 3n + i - 5$, $1 \le i \le n - 3$. We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge diam^D(G) + 1 = 3n - 2$ for every pair of vertices (u, v) where $u \ne v$.

**Case 1.** $v_i$ and $v_j$ where $|i-j| = 1$.

Subcase1. Without loss of generality suppose $i = 1$.

Then $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 4 + \left\lceil \frac{3n-7+3n+1}{2} \right\rceil \geq 3n-2$.

If both $v_i$ and $v_j$ are intermediate adjacent vertices, then

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 5 + \left\lceil \frac{3n+i-5+3n+j-5}{2} \right\rceil \geq 3n-2$.

**Case 2.** $v_i$ and $v_j$ where $|i-j| > 1$. If both $v_i$ and $v_j$ are end vertices,

Then $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 3n-3 + \left\lceil \frac{3n-7+3n+i-5}{2} \right\rceil \geq 3n-2$.

If either $v_i$ or $v_j$ (not both) is an end vertex, **t**hen

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 3n-5 + \left\lceil \frac{3n-7+3n+j-5}{2} \right\rceil \geq 3n-2$.

If both $v_i$ and $v_j$ are intermediate vertices, then

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 3n-7 + \left\lceil \frac{3n+i-5+3n+j-5}{2} \right\rceil \geq 3n-2$.

Therefore, $f(v_n) = 4(n-2)$ is the largest label.

Hence, $rmn^D(P_n) \leq \begin{cases} 2, & n = 2. \\ 4(n-2), & n \geq 3. \end{cases}$

**Theorem 2.4.**

The radio mean D-distance number of a star, $rmn^D(K_{1,n}) \leq \begin{cases} 2, & n = 1. \\ 4, & n = 2. \\ n+1, & n \geq 3. \end{cases}$

**Proof .**

It is obvious that $diam^D(K_{1,n}) = n + 4$. Let $V(K_{1,n}) = \{v_0, v_1, v_2, v_3, \ldots, v_n\}$, where $v_0$ is the central vertex. Define the function $f$ as $f(v_0) = n + 1$, $f(v_i) = i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = n + 5$ for every pair of vertices $(u, v)$ where $u \neq v$.

For $(v_0, v_i)$, $i = 1, 2, 3, \ldots, n$,

$d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq n + 2 + \left\lceil \frac{n+1+i}{2} \right\rceil \geq n + 5$.

For any pair$(v_i, v_j)$,

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 4 + \left\lceil \frac{i+j}{2} \right\rceil \geq n + 5.$

Therefore, $f(v_0) = n + 1$ is the largest label,

Hence, $rmn^D(K_{1,n}) \leq \begin{cases} 2, & n = 1. \\ 4, & n = 2. \\ n+1, & n \geq 3. \end{cases}$ □

The subdivision of a star $K_{1,n}$ denoted by $S(K_{1,n})$ is the graph obtained from $K_{1,n}$ by inserting a vertex an each edge of $K_{1,n}$ .

**Theorem 2.5.** The radio mean D-distance number of a subdivision of a star, $rmn^D S(K_{1,n}) \leq \begin{cases} 4, & n = 1. \\ 2(n+3), & n \geq 2. \end{cases}$

**Proof .**

It is obvious that $diam^D S(K_{1,n}) = n + 10$. Let $V(S(K_{1,n})) = \{v_0\} \cup \{v_i, u_i / i = 1, 2, \ldots, n\}$ and $E = \{v_0v_i, v_iu_i / i = 1, 2, 3, \ldots, n\}$. Define the function $f$ as $f(v_0) = 6$, $f(u_i) = n + 7 - i$, $1 \leq i \leq n$, $f(v_i) = n + 6 + i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = n + 11$, for every pair of vertices $(u, v)$ where $u \neq v$.

For $(v_0, u_i)$, $d^D(v_0, u_i) + \left\lceil \frac{f(v_0)+f(u_i)}{2} \right\rceil \geq n + 5 + \left\lceil \frac{6+n+7-i}{2} \right\rceil \geq n + 11.$

For $(v_0, v_i)$, $d^D(v_0, v_i) + \left\lceil \frac{f(v_0)+f(v_i)}{2} \right\rceil \geq n + 3 + \left\lceil \frac{6+n+6+i}{2} \right\rceil \geq n + 11.$

For any pair $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 6 + \left\lceil \frac{n+6+i+n+6+j}{2} \right\rceil \geq n + 11.$

For any pair $(u_i, u_j)$,

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq n + 10 + \left\lceil \frac{n+7-i+n+7-j}{2} \right\rceil \geq n + 11.$

For $u_i$ and $v_j$ where $|i - j| = 1$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 4 + \left\lceil \frac{n+7-i+n+6+i}{2} \right\rceil \geq n + 11.$

For $u_i$ and $v_j$ where $|i - j| > 1$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq n + 8 + \left\lceil \frac{n+7-i+n+6+j}{2} \right\rceil \geq n + 11.$

Therefore, $f(v_n) = 2(n + 3)$ is the largest label.

Hence, $rmn^D S(K_{1,n}) \leq \begin{cases} 4, n = 1. \\ 2(n+3), n \geq 2. \end{cases}$ □

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